RELATION BETWEEN COMPRESSIONAL WAVE VELOCITY AND AQUIFER POROSITY

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ABSTRACT

In recent years several formulas relating compressional wave velocity to the porosity of rock have been derived by different investigators. In this study, these formulas and data taken on the New Mexico Institute of Mining and Technology campus were used to determine porosity values for two aquifers.

Because each formula yielded different values of porosity for the range of velocities obtained in the field, independent information on porosity was needed. Pumping tests of two wells near the locality of the seismic measurements provided data through which average values of the specific yield of the aquifers was estimated. The specific retention of the aquifers was based on data for the South Coastal Basin aquifer in California. The latter is geologically similar to the aquifers studied.

By comparing calculated values of porosity with those obtained from the pumping tests, the formulas which gave the best value of porosity on the basis of compressional wave velocity were selected. Three were found to give reasonable values of porosity for the aquifers studied.

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RELATION BETWEEN COMPRESSIONAL WAVE VELOCITY AND AQUIFER POROSITY

INTRODUCTION

The seismic method has been used for oil exploration with extraordinary success for more than three decades. Because this method is expensive, very little application of this technique to water exploration has been attempted. In view of the expense, seismic surveys for water can be justified only if the method yields information other than merely the presence of water. One type of information which could be used to evaluate the potential water production from a given aquifer is the porosity.

This paper describes an evaluation of field measurements of compressional wave velocity through an aquifer as a means of determining its porosity. A seismic refraction survey was conducted in 1957-1958 by Dr. Allan Sanford along an 8,200 foot east-west trending line, located in the Rio Grande Valley near Socorro, New Mexico (Figure 1). The Rio Grande Valley is a fault-bounded structural depression filled with several thousand feet of Tertiary and Recent

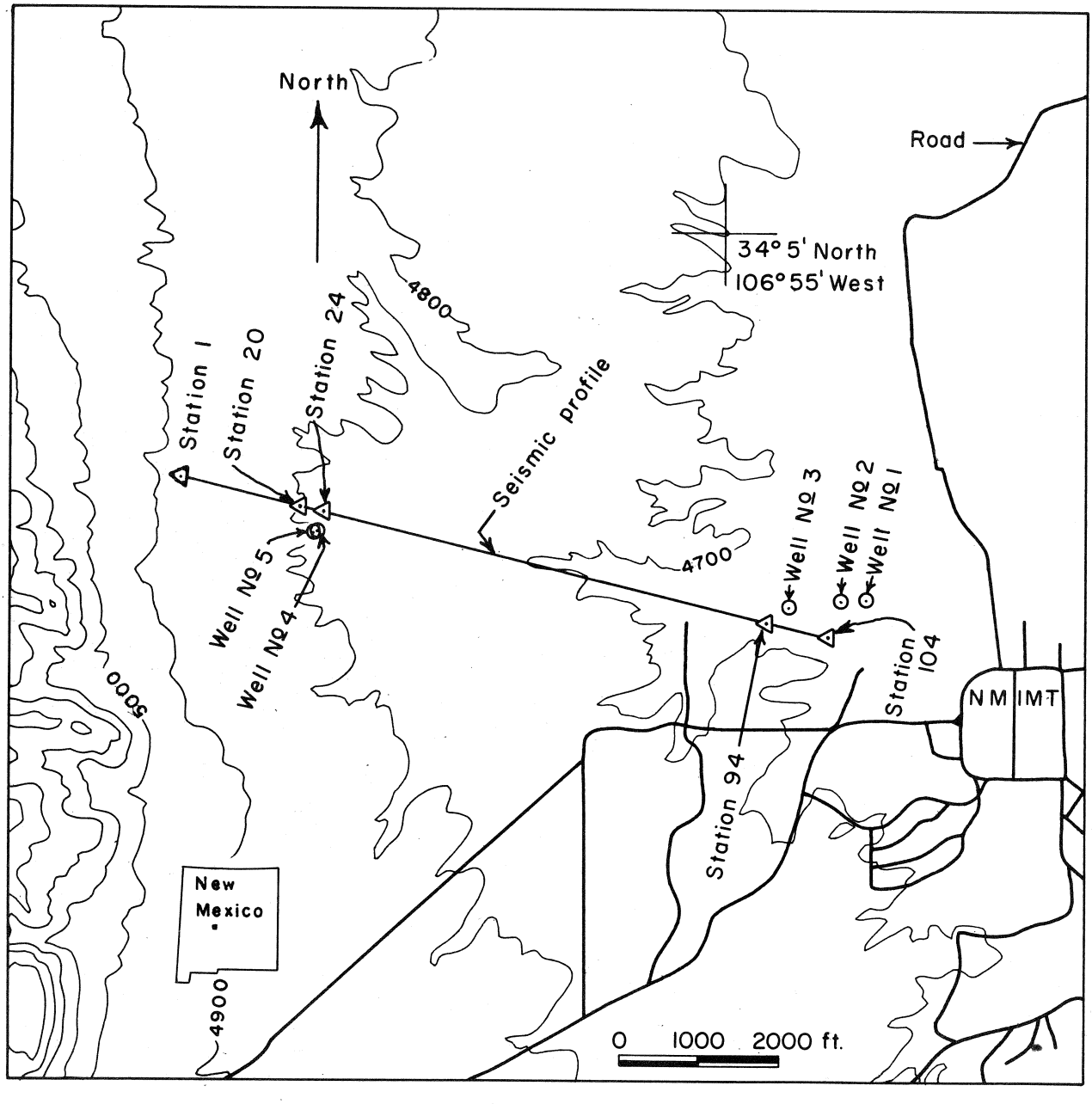


Fig. 1. Index map showing location of the area, seismic profile, and wells used in the tests.

sediments (Anderson, 1955). The main water table in the area corresponds to the level of the water in the Rio Grande. The water table declines about 8 inches per mile toward the south (Ground Water Project, 1958). The main aquifer in the area is of the unconfined type. Near the west end of the refraction line is a perched unconfined aquifer.

The data obtained from the seismic refraction survey provided velocity values for the material above and below the water tables. From these values, aquifer porosities were determined using several different relationships between porosity and velocity, based on the work of Biot (1956 a,b), Geertsma (1957), Brandt (1955), and Wyllie et al. (1956).

Since the pumping tests gave the specific yields of the aquifers and the seismic measurements gave the porosity, it was not possible to make a direct comparison between the seismic results and the well analyses. With reasonable assumptions for the specific retention of the aquifers, three theoretical porosity-velocity relationships gave reasonable values of porosity.

SEISMIC STUDY

Field Measurements

Velocities used for evaluating the porosities were obtained by the seismic refraction method (Dix, 1952). Refraction profiles were taken along an 8,200 foot line having 103 equally spaced stations (Figure 1). On the seismic records, the first arrivals (those which travel the minimum time path between the shot point and geophones) were timed in milliseconds (Figure 2). The time-distance curves for the refraction data are shown in Figure 3. The ordinate represents arrival time and the abscissa the distance of the geophones from the shot. Each time-distance curve has two linear segments with different slope. The first segment originates at the shot and has a slope of $1/V_d$. V_d is the velocity in the aerated layer lying between the surface and the water table. The arrivals for this segment of the curve travel directly from the shot to the geophones. The second segment intercepts the first at some distance, X_c, and has a slope of $1/V_w$. V_w is the velocity in the material beneath the water table. The arrivals for this segment of the curve are refracted arrivals (Dix, 1952).

The depths to the water table were found by using the unreversed profile method of Dix (1952, p. 260). The depths calculated by this

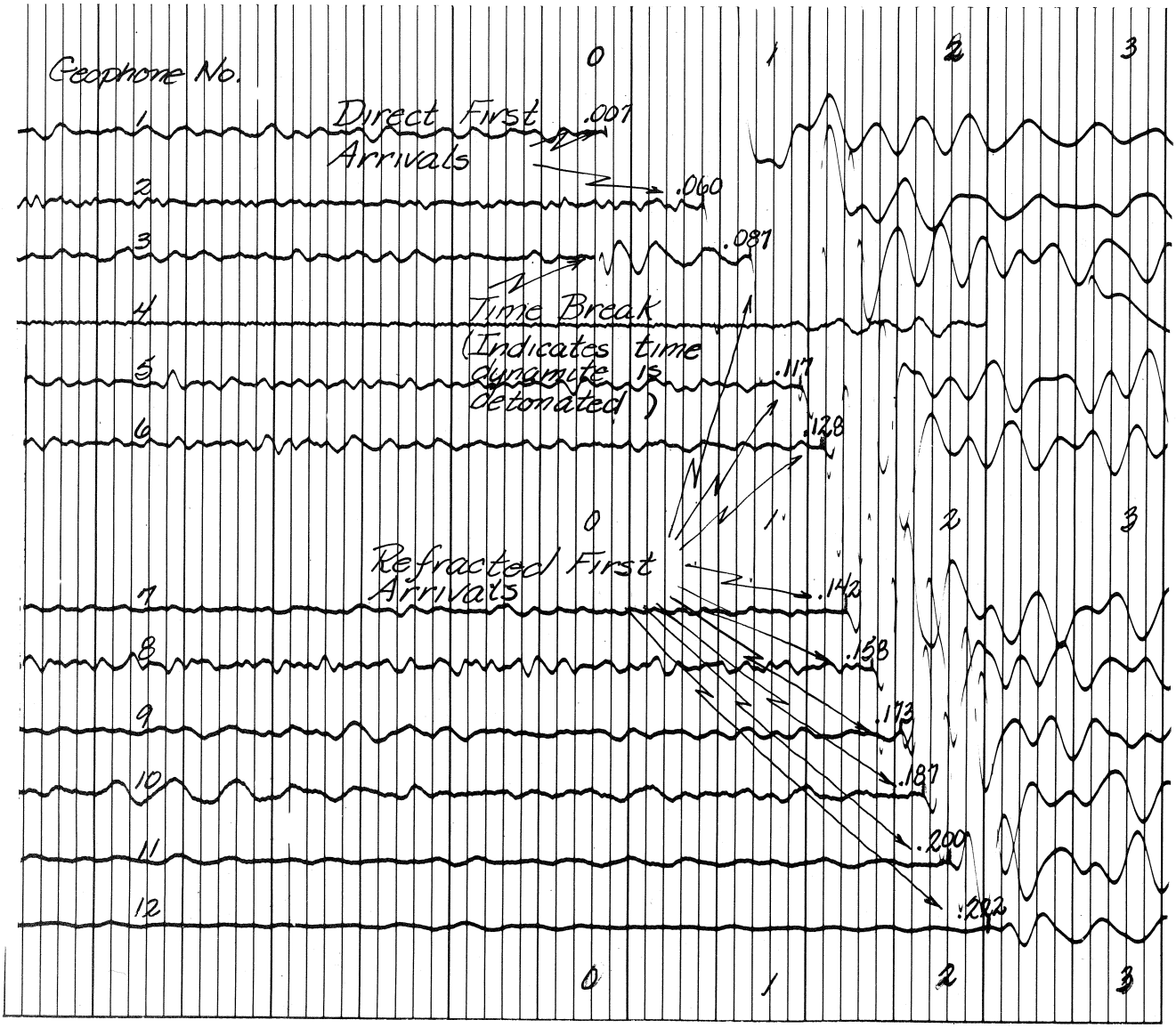
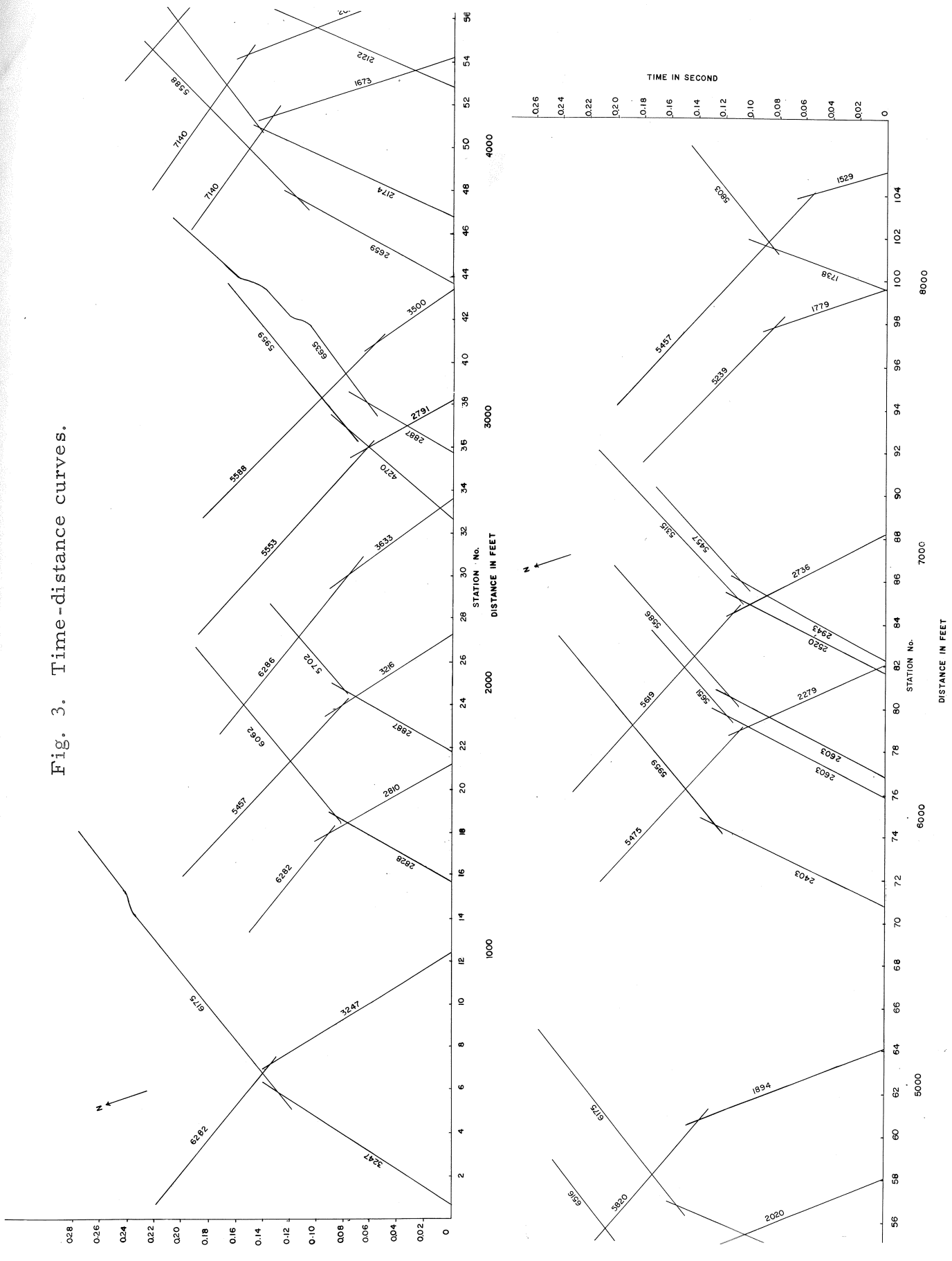


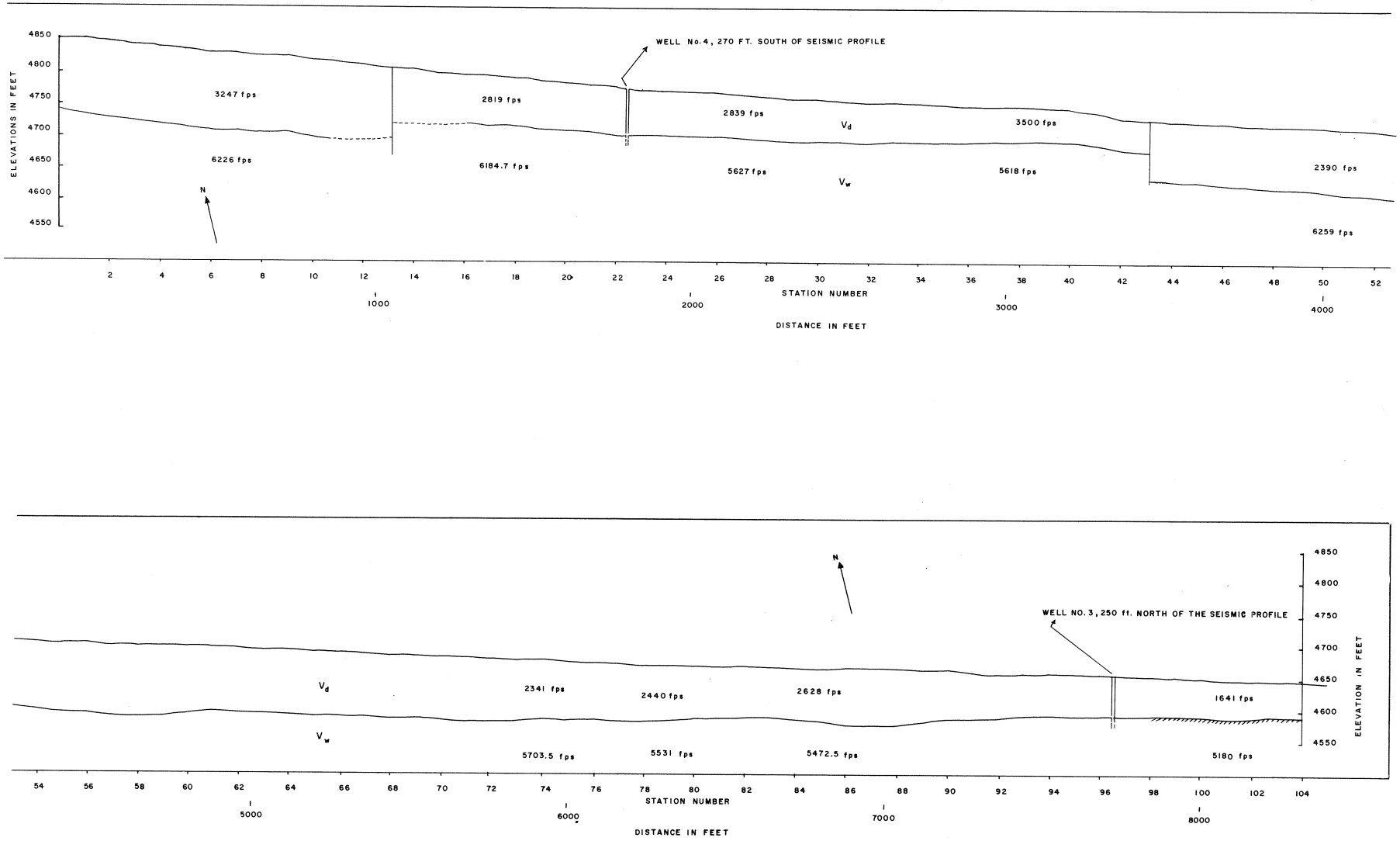
Fig. 2. An example of a seismic refraction record.


TIME IN SECOND
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
N
6175
3247
6282
3247
5588
7140
7140
5588
2122
1673
2174
2659
3500
5959
6635
2791
2887
4270
5553
3633
6286
5702
3216
2887
6062
5457
2810
6282
2828
1000
2000
3000
4000
STATION No.
DISTANCE IN FEET
TIME IN SECOND
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
N
6175
5820
6316
1894
2020
5457
5239
5803
1529
1738
1779
5475
2403
5969
5695
5986
5535
5457
2736
2943
2820
1595
2279
2603
2603
5000
6000
7000
8000
STATION No.
DISTANCE IN FEET

method agree closely with those measured at two wells. The measured depth of 72 feet at well No. 4 is identical with the calculated depth at stations 21 and 22. The measured depth of 72.70 feet at well No. 3 is within 8 feet of the calculated depth at stations 96 and 97. Figure 4 is a cross section along the line showing the depth to water levels.

Near the western end of the line, a zone of perched water exists. This zone terminates at a lateral velocity discontinuity near station 43. East of station 43, the water table mapped is the main water table for the Rio Grande Valley near the town of Socorro. The main water table probably extends beneath the perched water, but it was not detected in the refraction survey. Since the lateral velocity discontinuity near station 43 is sharp, this feature has been shown as a fault on the cross section.

Fig. 4. Depth profile and locations of wells No. 3 and No. 5.



Theory

General Theory of Biot

The two velocities measured in the refraction survey are for elastic waves traveling through porous material which has either air or water in the interstices. The general theory of propagation of elastic waves in a fluid saturated system has been presented by Biot (1956). He denoted the stress tensor for a cube of porous material as

$$\begin{pmatrix} \sigma_{xx} + \sigma & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} + \sigma & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} + \sigma \end{pmatrix}, \qquad (1)$$

where σ_{xx}, σ_{xy} (the first subscript denotes the direction of the stress and the second the plane on which the stress is acting), ... etc., represent the stresses acting on the solid part of each face of the cube and σ the stress acting on the fluid part of each face of the cube (other symbols are defined in the appendix). σ is related to the fluid pressure p in the pores by the relation

$$\sigma = -\beta' p , \qquad (2)$$

where β' is the fraction of the fluid area per unit cross section. If the material is assumed to be isotropic, the volume of fluid in a thin slab is always a fraction β' of the total volume. Therefore, β' is identical with the quantity usually designated as porosity, and equation

(2) can be written in the form,

$$\sigma = -\phi p . \tag{3}$$

If u_x, u_y, and u_z represent the displacement components of the solid and U_x, U_y, and U_z represent the displacement components of the fluid, the strain components of the solid can be expressed as

$$e_{xx} = \frac{\partial u_x}{\partial x}, \text{etc.} \quad ; \quad e_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}, \text{etc.} ; \tag{4}$$

and the strain components of the fluid can be expressed as

$$\epsilon_{xx} = \frac{\partial U_x}{\partial x} , \text{etc.} \tag{5}$$

The dilatations for the bulk material and the liquid are

$$e = e_{xx} + e_{yy} + e_{zz} , \tag{6}$$

$$\epsilon = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} . \tag{7}$$

If the seven stress components σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{yz}, σ_{zx}, and σ are linear functions of the seven strain components e_{xx}, e_{yy}, e_{zz}, e_{xy}, e_{yz}, e_{zx}, and ϵ, and the material is isotropic, then the stress-strain relations are

$$\begin{aligned}
\sigma_{xx} &= 2N e_{xx} + Ae + Q\epsilon , \\
\sigma_{yy} &= 2N e_{yy} + Ae + Q\epsilon , \\
\sigma_{zz} &= 2N e_{zz} + Ae + Q\epsilon , \\
\sigma_{xy} &= N e_{xy} , \ \sigma_{yz} = N e_{yz} , \ \sigma_{zx} = N e_{zx} , \\
\sigma &= Qe + R\epsilon .
\end{aligned} \tag{8}$$

The constants A and N correspond to Lame's two coefficients λ and μ, respectively. The coefficient R is a measure of the pressure on the fluid required to force a certain volume of the fluid into the aggregate while the total volume remains constant, and Q is a coupling factor between the volume change of the solid and that of the fluid.

The kinetic energy T of a system per unit volume [in generalized coordinates] (Lindsay, 1951) can be expressed

$$T = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \dot{q}_i \dot{q}_j \,. \qquad (9)$$

An expansion of equation (9) for the fluid saturated medium gives

$$2T = d_{11}\left[\left(\frac{\partial u_x}{\partial t}\right)^2 + \left(\frac{\partial u_y}{\partial t}\right)^2 + \left(\frac{\partial u_z}{\partial t}\right)^2\right] + 2d_{12}\left[\frac{\partial u_x}{\partial t}\frac{\partial U_x}{\partial t} + \frac{\partial u_y}{\partial t}\frac{\partial U_y}{\partial t} + \frac{\partial u_z}{\partial t}\frac{\partial U_z}{\partial t}\right.$$
$$\left. + d_{22}\left[\left(\frac{\partial U_x}{\partial t}\right)^2 + \left(\frac{\partial U_y}{\partial t}\right)^2 + \left(\frac{\partial U_z}{\partial t}\right)^2\right]. \qquad (10)$$

The coefficient d_{12} represents a mass coupling between fluid and solid, d_{11} represents the total effective mass of the solid, and d_{22} is the effective mass of the fluid, i.e.,

$$d_{11} = d_1 - d_{12} \text{ , and } \quad d_{22} = d_2 - d_{12} \qquad (11)$$

where d_1 (equal $(1-\phi)d_s$) and d_2 (equal ϕd_L) represent the mass per unit volume of aggregate of the solid and the liquid, respectively.

If q_x represents the total force acting on the solid part per unit

volume in the x-direction and Q_x the total force on the fluid per unit volume in the x-direction, then by Lagrange's equation (Lindsay, 1951)

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{u}_x}\right) = \frac{\partial^2}{\partial t^2}\left(d_{11}u_x + d_{12}U_x\right) = q_x \ , \tag{12}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{U}_x}\right) = \frac{\partial^2}{\partial t^2}\left(d_{12}u_x + d_{22}U_x\right) = Q_x \tag{13}$$

But since q_x and Q_x can be expressed as stress gradients,

$$q_x = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \ , \tag{14}$$

$$Q_x = \frac{\partial \sigma}{\partial x} \ , \tag{15}$$

equations (12) and (13) can be rewritten

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \frac{\partial^2}{\partial t^2}\left(d_{11}u_x + d_{12}U_x\right) , \tag{16}$$

$$\frac{\partial \sigma}{\partial x} = \frac{\partial^2}{\partial t^2}\left(d_{12}u_x + d_{22}U_x\right) . \tag{17}$$

If the stresses are expressed in terms of strains (see equation (8)), equations (16) and (17) become

$$2N\frac{\partial e_{xx}}{\partial x} + A\frac{\partial e}{\partial x} + Q\frac{\partial \epsilon}{\partial x} + N\frac{\partial e_{xy}}{\partial y} + N\frac{\partial e_{xy}}{\partial x} = \frac{\partial^2}{\partial t^2}\left(d_{11}u_x + d_{12}U_x\right), \tag{18}$$

$$Q\frac{\partial e}{\partial x} + R\frac{\partial \epsilon}{\partial x} = \frac{\partial^2}{\partial t^2}\left(d_{12}u_x + d_{22}U_x\right) . \tag{19}$$

If the strains are expressed in terms of displacements (see equation (4)), equations (18) and (19) become

$$N\nabla^2 u + \text{grad}\left[(N+A)e + Q\epsilon\right] = \frac{\partial^2}{\partial t^2}(d_{11}u + d_{12}U), \tag{20}$$

$$\text{grad}\,(Qe + R\epsilon) = \frac{\partial^2}{\partial t^2}(d_{12}u + d_{22}U) \tag{21}$$

Inasmuch as

$$\text{div}\ u = e, \qquad \text{and} \qquad \text{div}\ U = \epsilon \tag{22}$$

the divergence of equations (20) and (21) will be

$$\nabla^2(Pe + Q\epsilon) = \frac{\partial^2}{\partial t^2}(d_{11}e + d_{12}\epsilon), \tag{23}$$

$$\nabla^2(Qe + R\epsilon) = \frac{\partial^2}{\partial t^2}(d_{12}e + d_{22}\epsilon), \tag{24}$$

where $P = A + 2N$.

Solutions of equations (23) and (24) are of the form

$$e = C_1 \exp\left[i(x + Vt)\right] \tag{25}$$

$$\epsilon = C_2 \exp\left[i(x + Vt)\right] \tag{26}$$

By substituting expressions for $\nabla^2 e$, $\nabla^2 \epsilon$, $\frac{\partial^2 e}{\partial t^2}$, ... etc., obtained from equations (25) and (26) into equations (23) and (24), the following equations are obtained:

$$C_1 P + C_2 Q = C_1 V^2 d_{11} + C_2 V^2 d_{12} \qquad (27)$$

$$C_1 Q + C_2 R = C_1 V^2 d_{12} + C_2 V^2 d_{22} \qquad (28)$$

Dividing equations (27) and (28) by

$$H = P + R + 2Q,$$

and setting

$$P/H = \sigma_{11}, \quad R/H = \sigma_{22}, \quad Q/H = \sigma_{12}, \quad \text{where} \quad \sigma_{11} + 2\sigma_{12} + \sigma_{22} = 1$$

$$d_{11}/d = \alpha_{11}, \quad d_{22}/d = \alpha_{22}, \quad d_{12}/d = \alpha_{12}, \quad \text{where} \quad \alpha_{11} + 2\alpha_{12} + \alpha_{22} = 1$$

$$V_c^2 = H/d, \qquad Z = V_c^2/V^2,$$

gives

$$C_1 \sigma_{11} + C_2 \sigma_{12} = C_1 \alpha_{11} (1/Z) + C_2 \alpha_{12} (1/Z), \qquad (29)$$

$$C_1 \sigma_{12} + C_2 \sigma_{22} = C_1 \alpha_{12} (1/Z) + C_2 \alpha_{22} (1/Z). \qquad (30)$$

Multiplying these equations by Z gives

$$C_1(\sigma_{11}Z-\alpha_{11}) + C_2(\sigma_{12}Z-\alpha_{12}) = 0 \quad , \tag{31}$$

$$C_1(\sigma_{12}Z-\alpha_{12}) + C_2(\sigma_{22}Z-\alpha_{22}) = 0 \tag{32}$$

By eliminating C_1 and C_2 between equations (31) and (32), the following result is obtained:

$$(\sigma_{11}\sigma_{22}-\sigma_{12}^2)Z^2-(\sigma_{22}\alpha_{11}+\sigma_{11}\alpha_{22}-2\sigma_{12}\alpha_{12})Z + (\alpha_{11}\alpha_{22}-\alpha_{12}^2) = 0 \tag{33}$$

Letting

$$a^2=\frac{\sigma_{11}\sigma_{22}-\sigma_{12}^2}{\alpha_{11}\alpha_{22}-\alpha_{12}^2} \quad , \qquad b'=\frac{\sigma_{22}\alpha_{11}+\sigma_{11}\alpha_{22}-2\sigma_{12}\alpha_{12}}{\alpha_{11}\alpha_{22}-\alpha_{12}^2} \quad ,$$

equation (33) becomes

$$a^2Z^2-b'Z+1=0 \quad . \tag{34}$$

Solving equation (34) for Z gives

$$Z_{1,2}=\frac{b'\mp\sqrt{b'^2-4a^2}}{2a^2} \tag{35}$$

Since equation (35) has two roots, the two velocities are

$$V_1^2 = V_c^2 / Z_1 = H / d Z_1 \qquad (36)$$

$$V_2^2 = V_c^2 / Z_2 = H / d Z_2 \qquad (37)$$

V_1 is the velocity for the wave of the first kind and has the higher value, while V_2 is the velocity for the wave of the second kind and has the lower value (Biot, 1956a).

Evaluation of Elastic Constant H in Terms of Compressibilities

The procedure for determining the value of the elastic constant H in terms of compressibilities is given by Geertsma (1957). Assume first a porous material in which the compressibility effects of the pore liquid is neglected. If the pore volume V_p and the bulk volume V_b are functions of the hydrostatic stress $\bar{\sigma}$ and the hydrostatic pore tension p, the change in pore volume is given by

$$d V_p = \left(\frac{\partial V_p}{\partial \bar{\sigma}} \right)_p d\bar{\sigma} + \left(\frac{\partial V_p}{\partial p} \right)_{\bar{\sigma}} dp \ , \qquad (38)$$

and the change in the bulk volume is given by

$$d V_b = \left(\frac{\partial V_b}{\partial \bar{\sigma}} \right)_p d\bar{\sigma} + \left(\frac{\partial V_b}{\partial p} \right)_{\bar{\sigma}} dp \qquad (39)$$

If the system is assumed to be isotropic and homogeneous, and

$\bar{\sigma}$ and p increase by the same amount, then

$$\bar{\sigma} - p = \text{constant, or} \quad d\bar{\sigma} = dp \tag{40}$$

Under these conditions, only uniform dilatation can exist in the matrix, and

$$\frac{dV_b}{V_b} = \frac{dV_p}{V_p} = \frac{dV_r}{V_r} = C_r\, dp = C_r\, d\bar{\sigma} \tag{41}$$

where C is the compressibility and r refers to the matrix.

Assuming $\bar{\sigma}$ -p equals a constant and with the help of equation (41), the following is obtained from equations (38) and (39):

$$\frac{1}{V_p}\left(\frac{\partial V_p}{\partial \bar{\sigma}}\right)_p + \frac{1}{V_p}\left(\frac{\partial V_p}{\partial p}\right)_{\bar{\sigma}} = \frac{1}{V_b}\left(\frac{\partial V_b}{\partial \bar{\sigma}}\right)_p + \frac{1}{V_b}\left(\frac{\partial V_b}{\partial p}\right)_{\bar{\sigma}} = C_r \tag{42}$$

By using the reciprocal theorem (Love, 1927, p. 174) which states that "The whole work done by the forces of the first set acting over the displacements produced by the second set, is equal to the whole work done by the forces of the second set, acting over the displacements produced by the first," the following equation is obtained:

$$d\bar{\sigma}\left(\frac{\partial V_b}{\partial p}\right) dp = -\, dp \left(\frac{\partial V_p}{\partial \bar{\sigma}}\right) d\bar{\sigma} \,, \tag{43}$$

$$\left(\frac{\partial V_b}{\partial p}\right) = -\left(\frac{\partial V_p}{\partial \bar{\sigma}}\right) \,, \tag{44}$$

Using the results obtained in the equations above, equations (38) and (39)

can be split into a part for which $\bar{\sigma} - p$ is kept constant, and another part giving the influence of an additional change in $\bar{\sigma}$ or p.

$$\frac{dV_p}{V_p} = \left(-\frac{1}{V_p}\frac{\partial V_p}{\partial \bar{\sigma}} + \frac{1}{V_p}\frac{\partial V_p}{\partial p}\right) dp + \frac{1}{V_p}\frac{\partial V_p}{\partial \bar{\sigma}}\left(d\bar{\sigma} - dp\right) \tag{45}$$

$$\frac{dV_b}{V_b} = \left(\frac{1}{V_b}\frac{\partial V_b}{\partial \bar{\sigma}} + \frac{1}{V_b}\frac{\partial V_b}{\partial p}\right) dp + \frac{1}{V_b}\frac{\partial V_b}{\partial \sigma}\left(d\bar{\sigma} - dp\right) . \tag{46}$$

Denoting

$$\frac{1}{V_b}\left(\frac{\partial V_b}{\partial \bar{\sigma}}\right) = C_b \tag{47}$$

and with the [illegible] of equation (42), equation (46) can be rewritten

$$\frac{dV_b}{V_b} = C_r\, dp + C_b\,(d\bar{\sigma} - dp) = -(C_b - C_r)\,dp + C_b\, d\bar{\sigma} \tag{48}$$

If equation (39) is divided by V_b and the coefficient of the (dp) term is equated to the coefficient of the (dp) term in equation (48), the following result is obtained

$$\frac{1}{V_b}\left(\frac{\partial V_b}{\partial p}\right) = -\left(C_b - C_r\right) \tag{49}$$

Equation (49) and equation (44) give

$$\frac{1}{V_p}\left(\frac{\partial V_p}{\partial \bar{\sigma}}\right) = \frac{1}{\phi}\left(C_b - C_r\right) \tag{50}$$

Substituting equation (50) into equation (45) gives

$$\frac{dV_p}{V_p} = -\frac{1}{\phi}\left[C_b - (1+\phi)C_r\right]dp + \frac{1}{\phi}(C_b - C_r)\,d\bar{\sigma} \tag{51}$$

ξ represents the volume of fluid that enters the pores of a unit volume of bulk material. It can be expressed as ϕ ($e - \epsilon$) or as

$$\xi = \phi\,\frac{dV_p}{V_p} \tag{52}$$

Using equation (52) and the definition $e = dV_b/V_b$, equations (48) and (51) can be rewritten

$$e = -(C_b - C_r)p + C_b\bar{\sigma} \tag{53}$$

$$\xi = -\left[C_b - (1+\phi)C_r + \phi C_L\right]p + (C_b - C_r)\bar{\sigma} \tag{54}$$

where the compressibility of the fluid is now included in equation (54). Solving for $\bar{\sigma}$ from equations (53) and (54) gives

$$\bar{\sigma} = \frac{1 - B - \phi B + \phi B\,C_L/C_b}{(1-\phi-B)C_r + \phi C_L}\,e - \frac{1-B}{(1-\phi-B)C_r + \phi C_L}\,\xi \tag{55}$$

where $B = C_r/C_b$.

The hydrostatic stress of the bulk material is given by

$$\bar{\sigma} = \sigma_h + \sigma \tag{56}$$

where σ_h is the hydrostatic stress in the solid and is denoted by

$$\sigma_h = \frac{1}{3}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}) \tag{57}$$

Equation (56), after substituting for σ_h, can be rewritten

$$\bar{\sigma} = \frac{1}{3}(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})+\sigma \tag{58}$$

By substituting for σ_{xx}, σ_{yy}, σ_{zz}, and σ in terms of strains (equation (6)), equation (58) becomes

$$\bar{\sigma} = (P+Q-4/3N)\,e + (Q+R)\,\epsilon \tag{59}$$

Using the relation,

$$\xi = \phi\,(e-\epsilon) \tag{60}$$

equation (59) can be written

$$\bar{\sigma} = (H-4/3\,N)\,e - K\xi \tag{61}$$

where $K = (Q+R)/\phi$.

If the coefficient of the (e) term in equation (61) is equated to the coefficient of the (e) term in equation (55), the following result is obtained:

$$H - \frac{4}{3}N = \frac{1-B-\phi B+\phi B\,c_L/c_b}{(1-\phi-B)c_r+\phi c_L} \tag{62}$$

$$H = \frac{(1-B)^2}{(1-\phi-B)c_r+\phi c_L} + \left(\frac{B}{c_r}+\frac{4}{3}N\right)$$

Therefore equation (36) can be rewritten

$$V_1^2 = \left[\frac{(1-B)^2}{(1-\phi-B)c_r+\phi c_L} + \left(\frac{B}{c_r} + \frac{4}{3} N \right) \right] \Big/ d Z_1 \qquad (63)$$

Application of General Theory to Special Cases

If the dilatation of the solid and the liquid are equal, i.e. $e = \epsilon$, the amplitudes C_1 and C_2 of equations (25) and (26) are equal. Therefore, equations (31) and (32) become equal and

$$Z = \frac{\sigma_{11}+\sigma_{12}}{\alpha_{11}+\alpha_{12}} = \frac{\sigma_{22}+\sigma_{12}}{\alpha_{22}+\alpha_{12}} = \frac{\sigma_{11}+2\sigma_{12}+\sigma_{22}}{\alpha_{11}+2\alpha_{12}+\alpha_{22}} = 1 \qquad (64)$$

Hence equation (63) becomes

$$V_1^2 = \left[\frac{(1-B)^2}{(1-\phi-B)c_r+\phi c_L} + \left(\frac{B}{c_r} + \frac{4}{3} N \right) \right] \Big/ d \qquad (65)$$

The second term in the numerator, $(B/C_r + 4/3N)$, is equal to $(3/C_b)$ $(1-\nu/1+\nu)$ (ν is Poisson's ratio) and the denominator d is equal to $((1-\phi)d_r + \phi d_L)$. Therefore equation (65) can be written

$$V_1^2 = \left[\frac{(1-B)^2}{(1-\phi-B)c_r+\phi c_L} + \frac{3}{c_b}\left(\frac{1-\nu}{1+\nu} \right) \right] \Big/ \left[(1-\phi) d_r + \phi d_L \right] \qquad (66)$$

In a dry system in which the pores are filled with air, the air density and bulk modulus are negligible. Therefore, the compressibility of the dry system is equal to the bulk compressibility C_b. For the dry system, the equation for longitudinal wave velocity can be written

$$V_d^2 = \frac{3\,(1-\nu')}{(1+\nu)\,(1-\phi)\,d_r\,C_b} \tag{67}$$

If C_b is eliminated between equations (67) and (66), (the equation for a water-saturated system), the following result is obtained:

$$V_w^2 = \frac{\left[\dfrac{\left(1-\dfrac{C_r\,(1+\nu')\,(1-\phi)\,d_r\,V_d^2}{3(1-\nu')}\right)^2}{\left(1-\phi-\dfrac{C_r\,(1+\nu')\,(1-\phi)\,d_r\,V_d^2}{3(1-\nu')}\right)C_r+\phi\,C_L}+\dfrac{3(1-\nu)\,(1+\nu')\,(1-\phi)\,d_r\,V_d^2}{3(1+\nu)\,(1-\nu')}\right]}{(1-\phi)\,d_r+\phi\,d_L} \tag{68}$$

By assuming that Poisson's ratio is approximately the same for the dry and the water-saturated systems, equation (68) can be written

$$V_w^2 = \left[\frac{\left(1-\dfrac{C_r\,(1+\nu)\,(1-\phi)\,d_r\,V_d^2}{3(1-\nu)}\right)^2}{\left(1-\phi-\dfrac{C_r\,(1+\nu)\,(1-\phi)\,d_r\,V_d^2}{3(1-\nu)}\right)C_r+\phi\,C_L}+(1-\phi)\,d_r\,V_d^2\right] \Big/ \left[(1-\phi)\,d_r+\phi\,d_L\right] \tag{69}$$

Since the second term $\frac{C_r(1+\nu)(1-\phi)d_r V_d^2}{3(1-\nu)}$ in the numerator is very small with respect to the first term, it can be neglected. The same term in the denominator can also be neglected. Therefore, equation (69) can be written

$$V_w^2 = \left[\frac{1}{(1-\phi)C_r+\phi C_L} + (1-\phi)d_r V_d^2\right] \Big/ \left[(1-\phi)d_r+\phi d_L\right] \qquad (70)$$

Now if the matrix compressibility (C_r) for the water-saturated system is considered zero, equation (66) becomes

$$V_w^2 = \left[\frac{3(1-\nu)}{(1+\nu)C_b} + \frac{1}{\phi C_L}\right] \Big/ \left[(1-\phi)d_r+\phi d_L\right] \qquad (71)$$

Equation (71) is known as the Laughton equation (Geertsma and Smit, 1961).

Eliminating C_b between equations (71) and (67) after putting $\nu = \nu' = 1/2$, the following result is obtained:

$$V_w^2 = \left[(1-\phi)d_r V_d^2 + \frac{1}{\phi C_L}\right] \Big/ \left[(1-\phi)d_r+\phi d_L\right] \qquad (72)$$

Equation (72) is identical to equation (70) except that the matrix compressibility (C_r) is zero.

For a suspension, $C_b = \infty$ (Geertsma and Smit, 1961), and

equation (66) for a water-saturated system becomes

$$V_w^2 = \frac{1}{\left[(1-\phi)\, d_r + \phi d_L\right]\left[(1-\phi)\, C_r + \phi C_L\right]} \qquad (73)$$

Equation (73) is known as Wood's formula (Wood, 1941).

Brandt's Formula for a Liquid Saturated System

Another important formula which applies to nonspherical particles in a system with variable porosity is presented by Brandt (1955). His formula for a liquid-saturated sandstone is

$$V_w^2 = \frac{2\left(\frac{1-\nu_L}{1+\nu_L}\right)\left(P-\bar{C}P_L\right)^{1/3}}{3\phi\left[(1-\phi)\, d_r + \phi\, d_L\right]\bar{K}^{2/3}} \quad \frac{\left[1+17.5\; C_L^{-3/2}\,\bar{K}\left(P-\bar{C}P_L\right)^{-1/2}\right]^{5/3}}{\left[1+26.3\; C_L^{-3/2}\,\bar{K}\left(P-\bar{C}P_L\right)^{-1/2}\right]} \qquad (74)$$

If P approximately equals P_L, then the term $(P-\bar{C}P_L)$ approaches zero; if $P-\bar{C}P_L$ approaches zero, the second term in both the numerator and denominator becomes very large compared to the first term. If the first term in both the numerator and denominator is neglected, equation (74) becomes

$$V_w^2 = \frac{3\left(\frac{1-\nu_L}{1+\nu_L}\right)\left(P-\bar{C}P_L\right)^{1/3}}{3\phi\left[(1-\phi)\, d_r + \phi\, d_L\right]} \quad \frac{(17.5)^{5/3}\, C_L^{-5/2}\,\bar{K}^{5/3}\left(P-\bar{C}P_L\right)^{-5/6}}{(26.3)\, C_L^{-3/2}\,\bar{K}\left(P-\bar{C}P_L\right)^{-1/2}}$$

$$V_w^2 = \frac{3(1-\nu_L)}{(1+\nu_L)\,\phi\, C_L\left[(1-\phi)\, d_r + \phi\, d_L\right]}, \qquad (75)$$

When $C_b = \infty$ in Laughton's formula, it is identical to equation (75) if $\nu_l = 1/2$. When $C_r = 0$ in Wood's formula, it is identical to equation (75) if $\nu_l = 1/2$.

Time-Average Formula

An equation which relates the system velocity to the velocity of its constituents is given by Wyllie et al. (1956) in the form

$$\frac{1}{V} = \frac{\phi}{V_p} + \frac{1-\phi}{V_s} \tag{76}$$

where V is the velocity of the system;

V_p is the velocity in the pore filler;

V_s is the velocity in the matrix material.

This equation, called the time-average formula, assumes that the system can be represented by a series of alternating layers of its constituents.

If the time-average formula is applied to both the aerated and water-saturated layers, assuming the material in the two layers is identical, it can be written in the form

$$\phi = \frac{\frac{1}{V_d} - \frac{1}{V_w}}{\frac{1}{V_{air}} - \frac{1}{V_{liquid}}} \tag{77}$$

Calculations and Results

In this section, velocities obtained in the field and porosity calculated from the theoretical relationships are presented. The range of velocity values used in the calculations corresponds to the range of seismic velocities measured in the field. For the aerated layer, the velocity values range from 1500 ft/sec to 4000 ft/sec, for the water-saturated layer the velocity values range from 5000 ft/sec to 6500 ft/sec.

The values of compressibilities and densities used in the calculation are listed below:

$C_r = 2.7 \times 10^{-12}$ cm^2/dyne, $\quad d_r = 2.65$ gm/cc

$C_L = 45.8 \times 10^{-12}$ cm^2/dyne, $\quad d_L = 1.00$ gm/cc

The sedimentary rocks in the area of refraction survey contain principally quartz, feldspar, and clay. Inasmuch as the densities and compressibilities of these constituents are nearly the same (Birch et al., 1936), a single value of C_r and d_r could be used in the calculations.

Theoretical Curves for the Water-Saturated Layer

Wood's formula.

Figure 5 gives the relation between porosity and velocity in the water-saturated layer on the basis of Wood's formula (equation (73)).

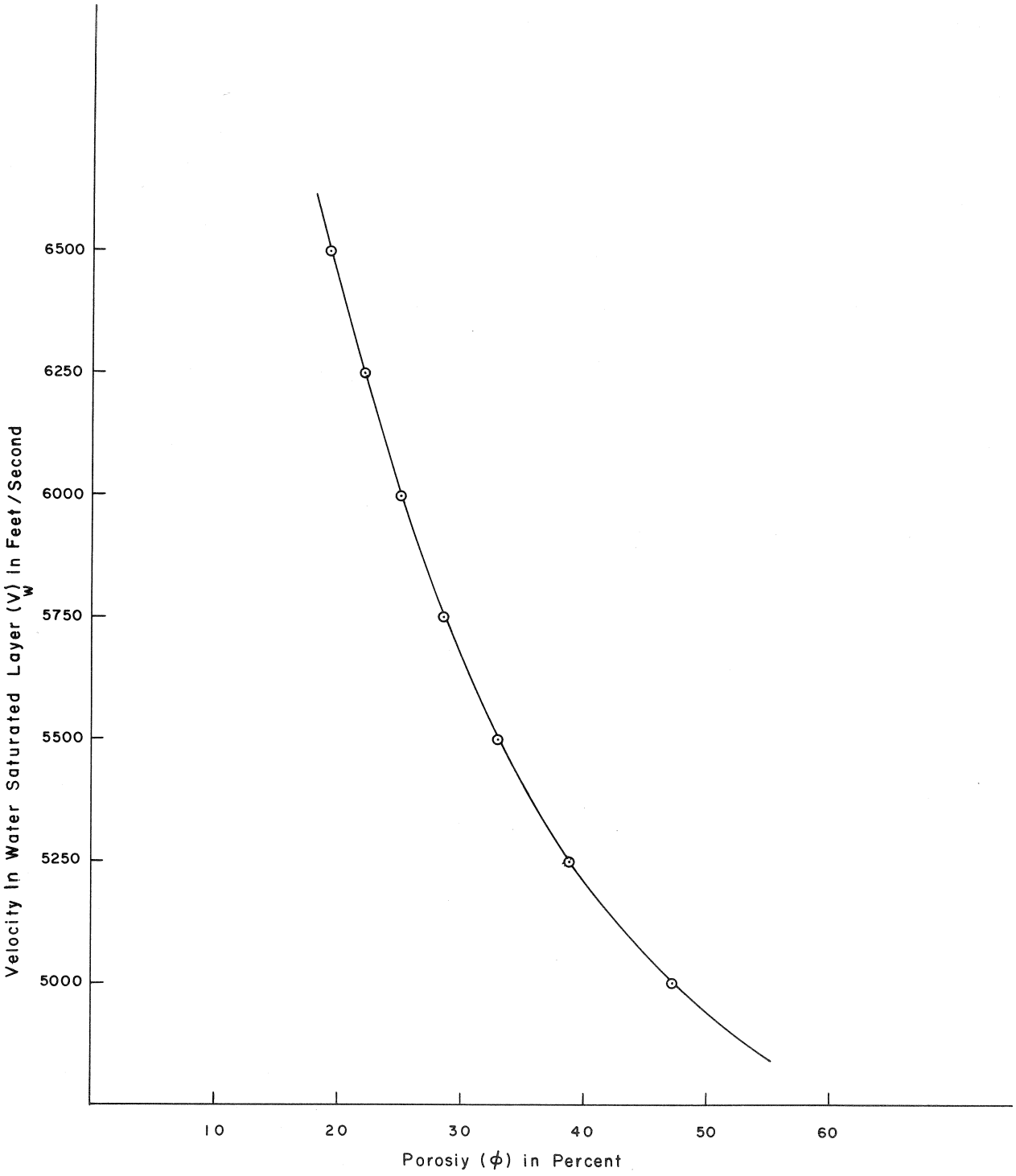


Fig. 5. Compressional wave velocity in the water-saturated layer versus porosity using Wood's formula.

This equation assumes that the water-saturated layer behaves like a suspension, i.e. $C_b = \infty$.

Modified Brandt's formula.

Figure 6 gives the relation between porosity and velocity in the water-saturated layer on the basis of the modified Brandt's formula (equation (75)). This equation assumes that the pressure of the liquid and the pressure on the solid are approximately the same.

Time-average formula.

Figure 7 gives the relation between porosity and velocity on the basis of the time-average formula (equation (76)). This equation assumes that the water-saturated layer can be represented by a series of alternating solid and liquid layers.

Theoretical Curve for the Aerated Layer

Time-average formula.

Figure 8 gives the relation between porosity and velocity in the aerated zone on the basis of the time-average formula, (equation (76)).

Theoretical Curves for Water-Saturated and Aerated Layers

If the material above and below the water table is assumed to be identical in all respects, except for the difference of fluid in the pores, then two velocities are available for porosity calculations.

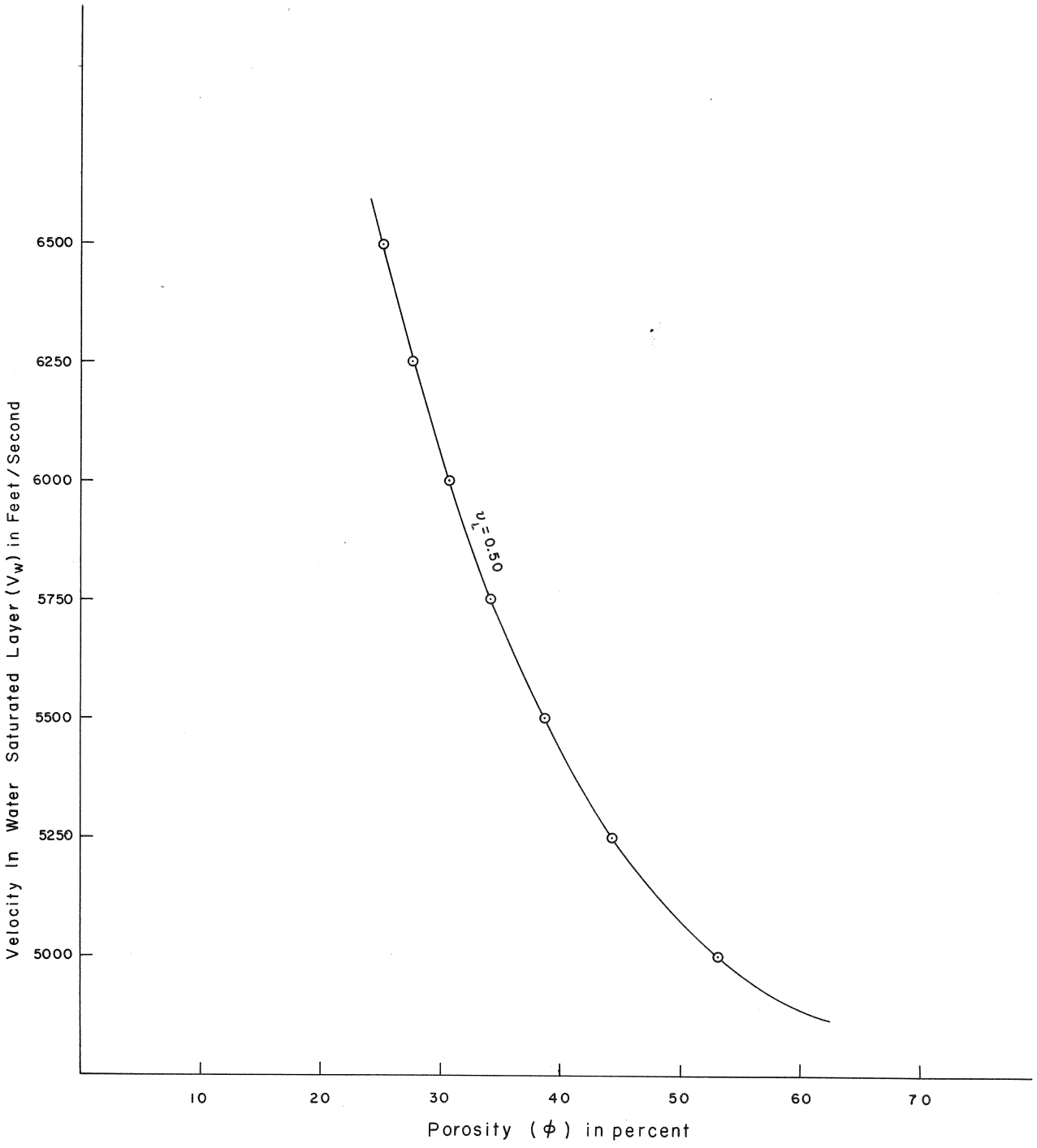


Fig. 6. Compressional Wave Velocity in the water-saturated layer versus porosity using modified Brandt's formula.

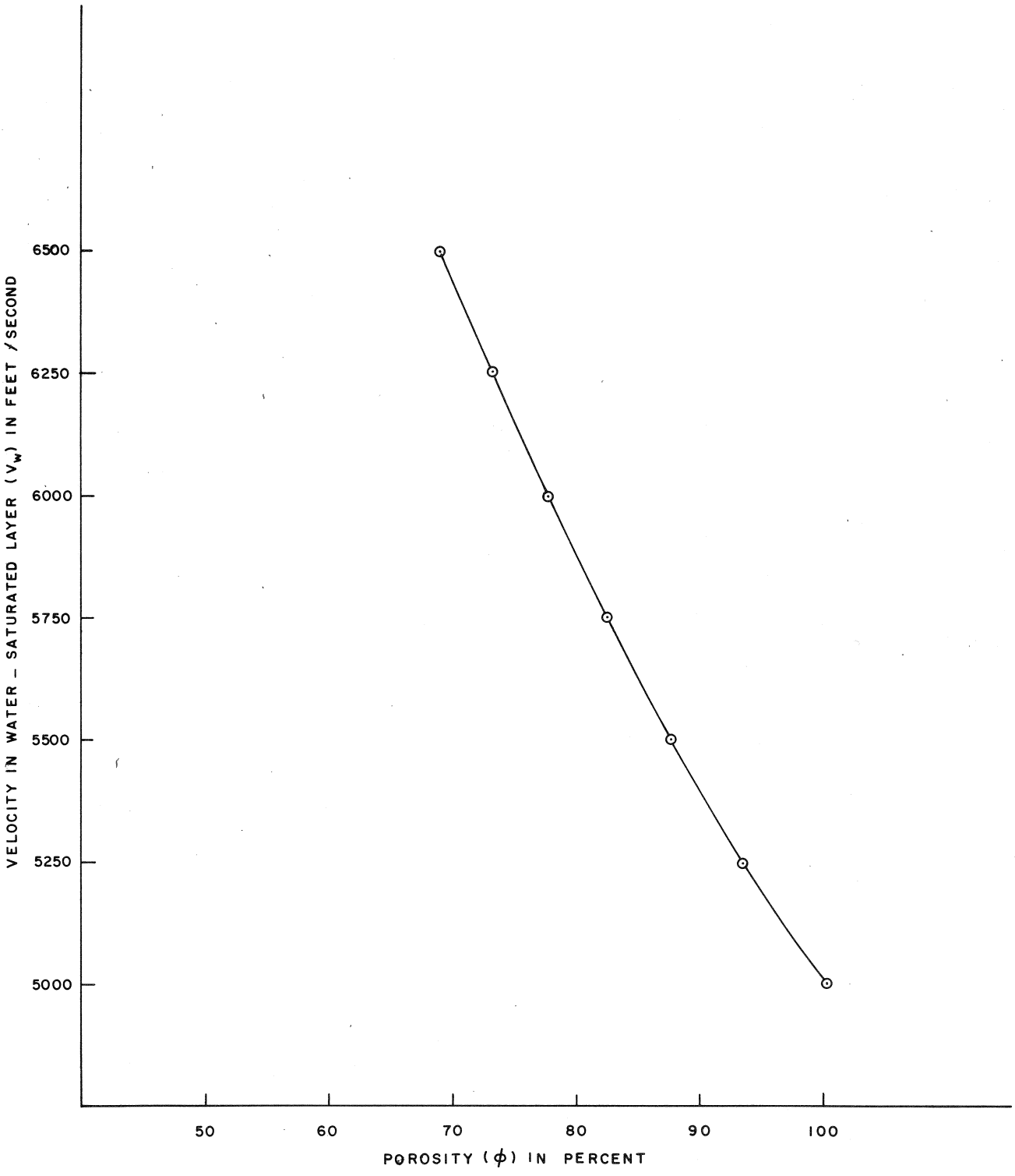


Fig. 7. Compressional wave velocity in the water-saturated layer versus porosity using time-average formula.

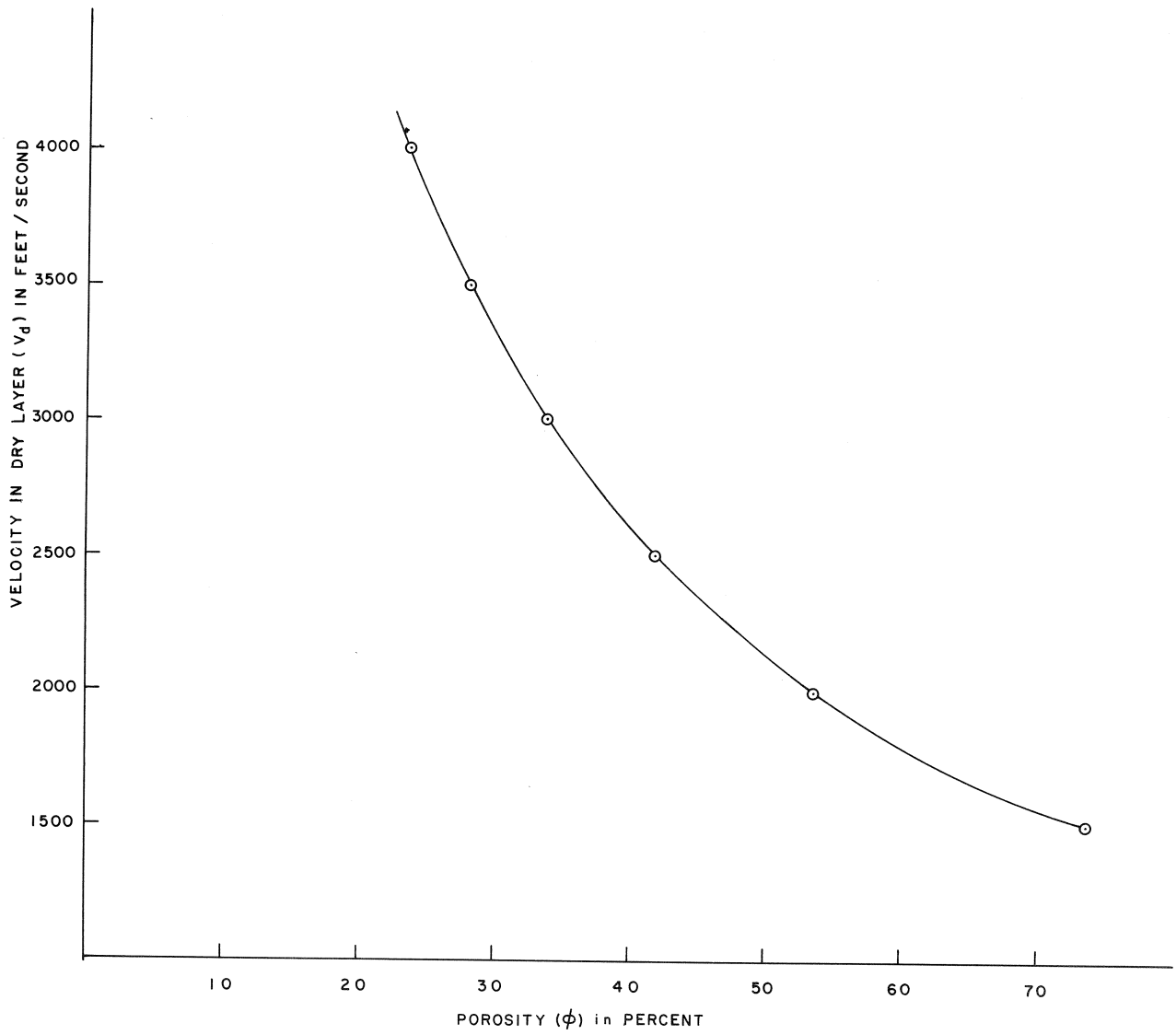


Fig. 8. Compressional wave velocity in the aerated layer versus porosity using time-average formula.

Applying equation (70).

Figure 9 gives the relation between porosity and velocity on the basis of equation (70), using the velocities in the aerated and water-saturated layers.

Applying equation (72).

Figure 10 gives the relation between porosity and velocity on the basis of equation (72), using the velocities in the aerated and water-saturated layers. This equation assumes that the matrix compressibility is zero.

Time-average formula for two layers case.

Figure 11 gives the relation between porosity and velocity on the basis of equation (77), using the velocities in the aerated and water-saturated layers.

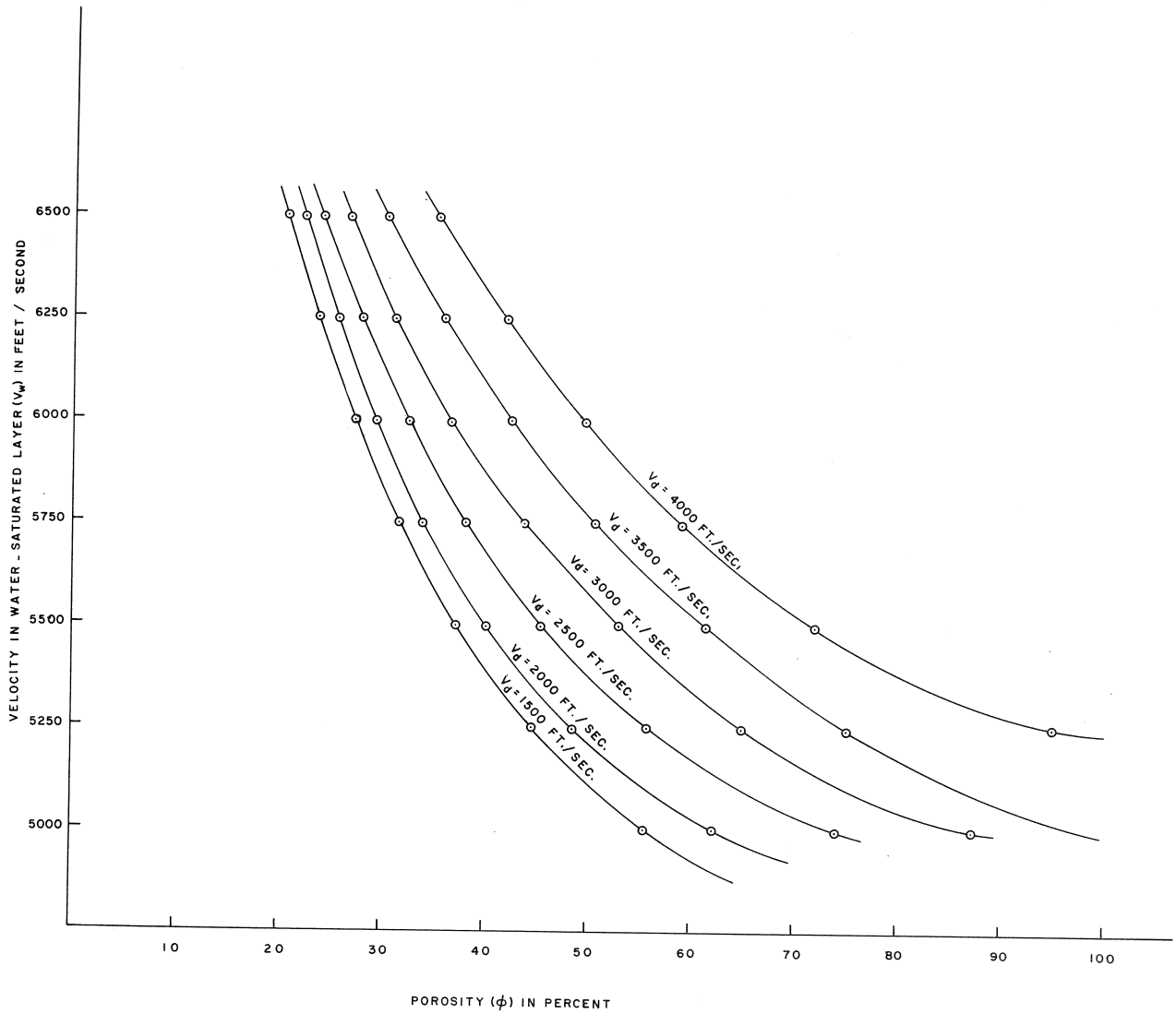


Fig. 9. Compressional Wave velocity in the aerated and water-saturated layers versus porosity using equation (70). The porosity in the aerated and water-saturated layers is the same.

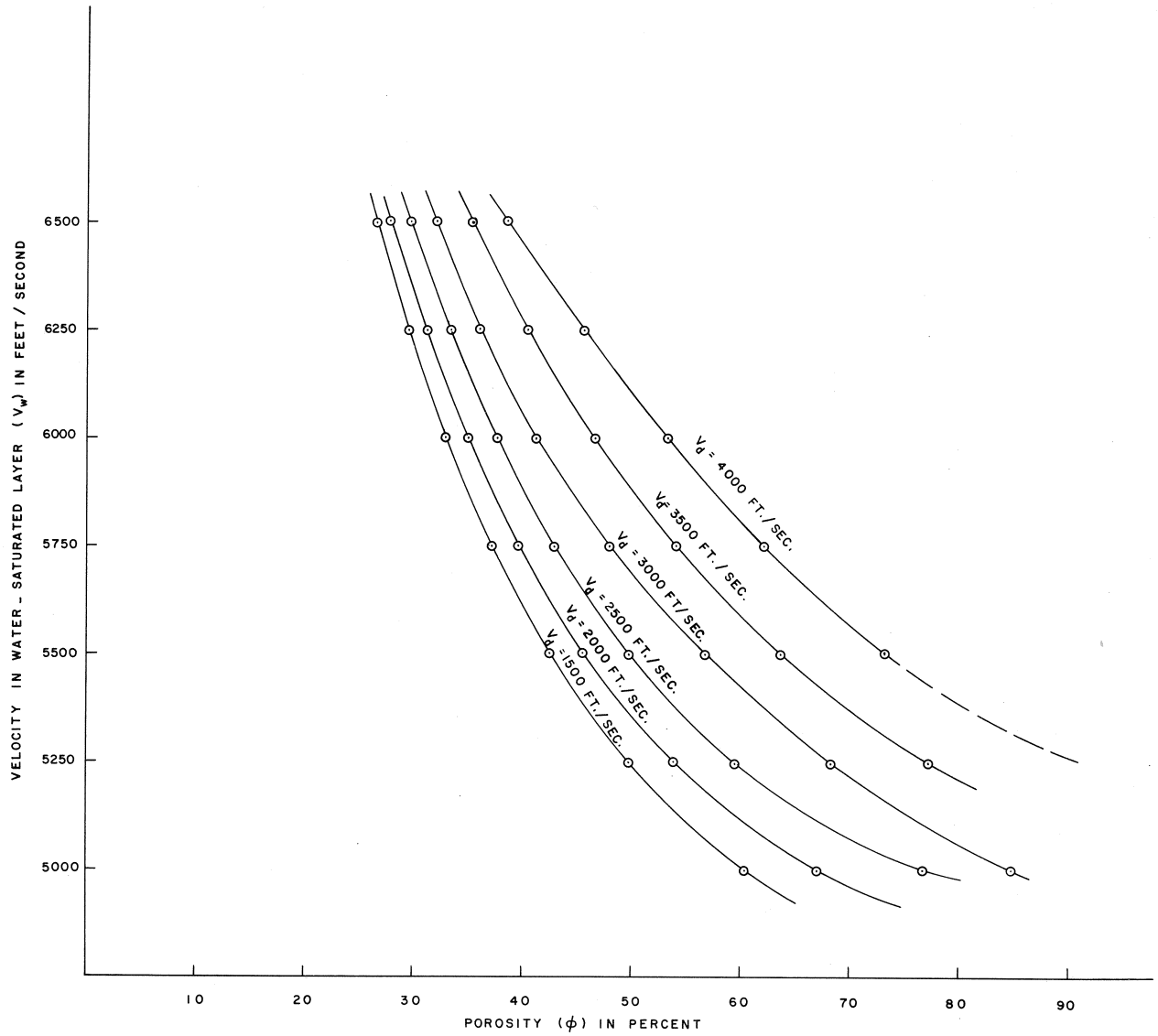


Fig. 10. Compressional wave velocity in the aerated and water-saturated layers versus porosity using equation (72). The porosity in the aerated and water-saturated layers is the same.

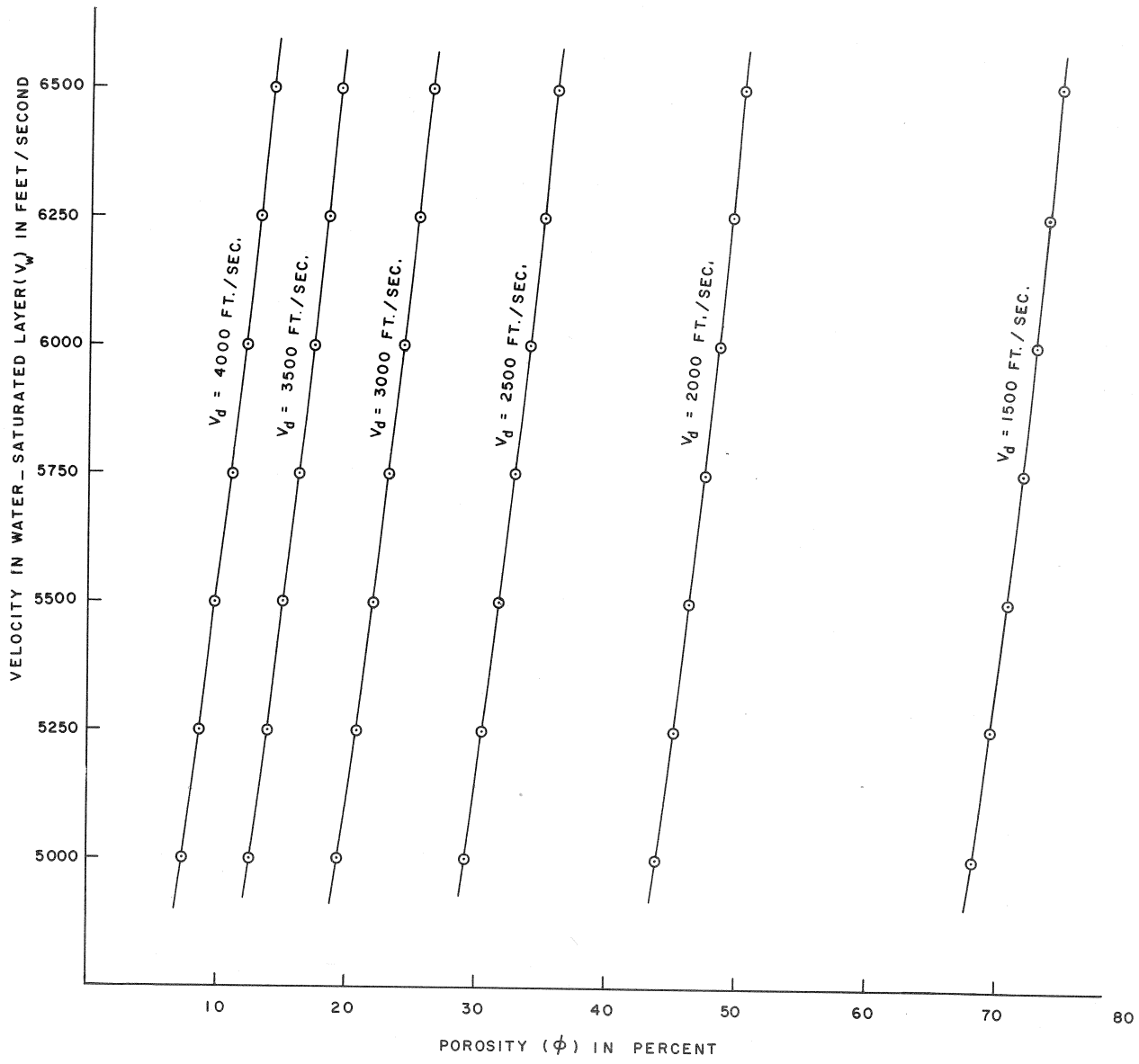


Fig. 11. Compressional wave velocity in the aerated and water-saturated layers versus porosity using time-average formula. The porosity in the aerated and water-saturated layers is the same.

HYDRAULIC PROPERTIES OF THE AQUIFERS

Introduction

To ascertain whether any of the relationships between velocity and porosity yield satisfactory porosity values, hydraulic properties of the aquifer were determined at two locations along the seismic profile.

Two pumping tests were conducted. The locations of the wells used in these tests with respect to the seismic profile is shown in Figure 1. In the first pumping test, the distance between the pumped well No. 1 and the observation wells No. 2 and No. 3 is 308 feet and 920 feet, respectively. In the second test, the distance between the pumped well No. 4 and the observation well No. 5 is 31 feet. The five wells in the area are partially penetrating the aquifers. The aquifer (a permeable geologic formation in which ground water occurs) in which the first test was conducted is a water-table (unconfined) aquifer. The aquifer in which the second test was conducted is a perched water-table aquifer. Figure 12 shows examples of these two aquifers.

In the first test, well No. 1 was pumped continuously at a constant rate of 300 gal/min. for 18 days and was observed for a recovery period of 13 days. In the second test well No. 4 was pumped continuously at a rate of 5.6 gal/min. for 8 hours. By analysing the data from the two tests, the specific yields of the two aquifers were obtained.

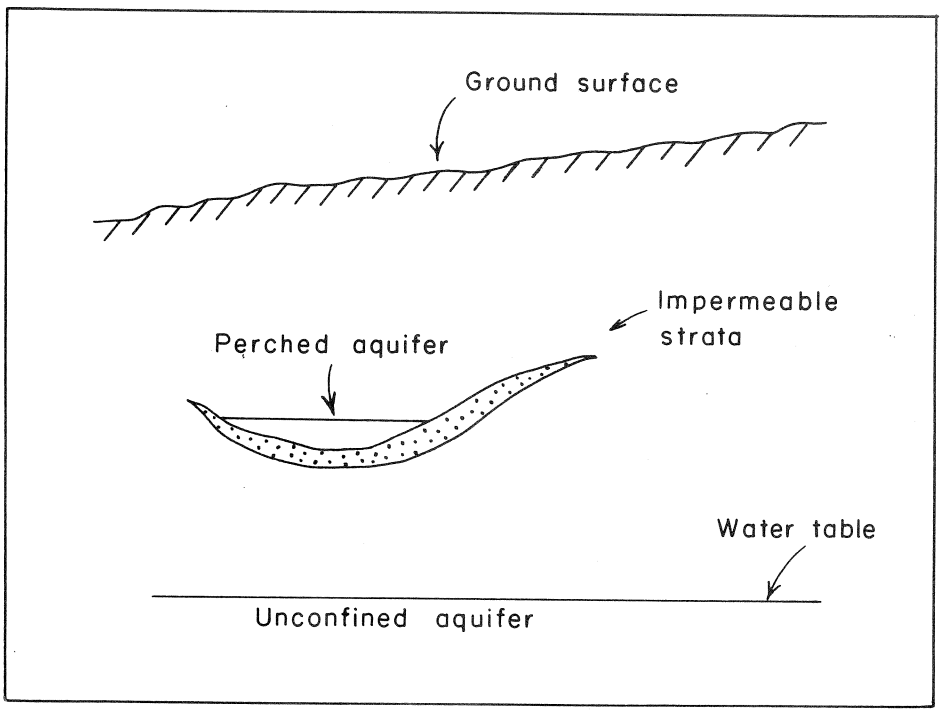


Fig. 12. Types of aquifers in the area.

The specific yield is the fractional volume of pore fluid drainable by gravity flow. The specific retention is the fractional volume of fluid which is retained in the aquifer because of adhesion to grain boundaries. In case of complete saturation, porosity is related to specific yield and specific retention by the following relation:

Porosity = specific yield + specific retention.

The specific yield and the specific retention depend to a large extent on the size of the grains in the aquifer. Figure 13 shows in a graphical form the relationship between grain size and specific yield and specific retention for the aquifer in the South Coastal Basin, California (after Eckis). As the grain size becomes smaller, the difference in porosity and specific yield increases.

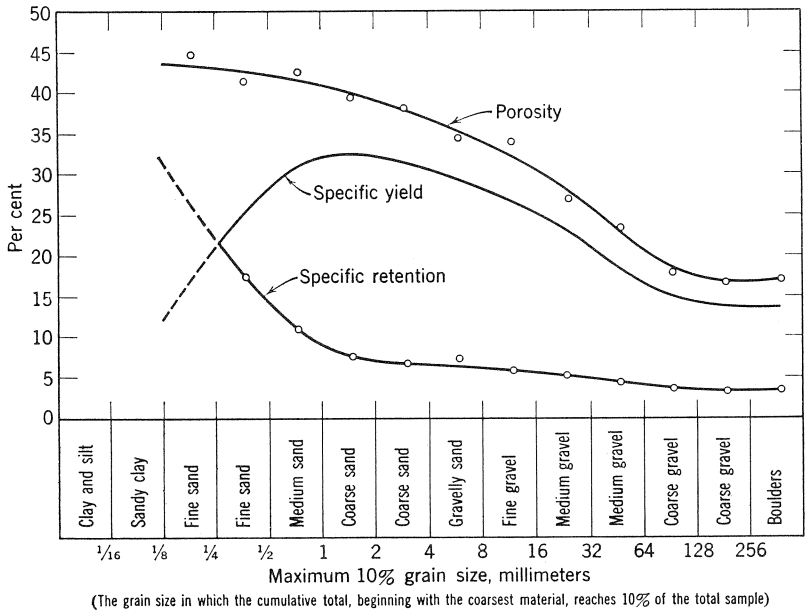


Fig. 13. Porosity, specific yield and specific retention (after Eckis).

Theory

If the pumping rate of a well completely penetrating an aquifer infinite in areal extent is constant, the unsteady equation for the drawdown at an observation well at a distance r from the pumping well is (Todd, 1959):

$$s = \frac{Q}{4\pi T} \int_{r^2 S/4Tt}^{\infty} \frac{e^{-y}\, dy}{y} = \frac{Q}{4\pi T} W(u) \qquad (78)$$

where W(u) = the well function;

s = the drawdown, L;

Q = the well discharge, L^3/T;

T = the coefficient of transmissibility, L^2/T;

S = the storage coefficient;

u = $r^2 S/4Tt$;

t = the time since pumping started, T.

Equation (78) is known as Theis's formula. A graphic solution of equation (78) has been described by Theis (Todd, 1959) for determining the formation coefficients. The Theis formula can be applied when the following conditions are more or less satisfied (Todd, 1959):

1. The aquifer is homogeneous and isotropic.
2. The aquifer is of infinite areal extent.

3. The well penetrates the whole aquifer.

4. The well diameter is infinitesimal.

5. The water removed from storage is discharged instantaneously with decline of head.

For a well which partially penetrates an aquifer whose thickness (b) is greater than (r) i.e. $r/b < 1$, the equation of the drawdown during the period $t < \frac{(2b-\ell)^2 S_s}{20K}$ (assuming that conditions 1, 2, 4, and 5 above apply) is given by Hantush (1961) as

$$s = \frac{Q}{4\pi K \ell}\left[M\left(u, \frac{\ell}{r}\right) - M\left(u, \frac{d}{r}\right)\right] \qquad (79)$$

where

K = the hydraulic conductivity of the aquifer, (volume of water that crosses a unit cross section of the aquifer per unit time, under a hydraulic gradient of 100%), L/T;

d = the depth from the top of the aquifer for the unscreened portion of the pumped well, L;

$M(u, \beta)$ is an infinite integral, tabular values for which are available (Hantush, 1961).

A graphic solution of equation (79) described by Hantush (1961) results in calculating the formation constants and, in many cases, the thickness of

the aquifer. The formula used for obtaining the thickness of the aquifer (Hantush, 1961) is

$$b = \frac{1}{2}\left[\ell + \frac{1}{2}(\ell' + d') + r\sqrt{5/u_d}\right] \qquad (80)$$

where

ℓ' = the depth of penetration of the observation well, L;

d' = the depth from the top of the aquifer of the unscreened portion of the observation well, L;

U_d = the point of departure of the observation curve from the type curve.

When (r) is greater than the thickness (b) of the aquifer, i.e., $r/b > 1$, the equation of the drawdown for a partially penetrating well has been shown by Hantush (1957) to be identical with Theis's formula; namely, equation (78).

The equation of drawdown around a well partially penetrating an infinitely thick aquifer is (Boulton, 1954):

$$s = \frac{Q}{2\pi K \ell}\left[\sinh^{-1}\frac{1}{\rho} - \sinh^{-1}\frac{1+T'}{\rho} + \sinh^{-1}\frac{T'}{\rho}\right], \qquad (81)$$

where

$$\rho = r/\ell \quad , \quad \text{and } T = Kt/S\ell \quad .$$

If the well is pumped for a short period of time, equation (81) becomes

(Hantush, 1960)

$$s = \frac{Qt}{2\pi S l r} \qquad (82)$$

Analyses of Data

The five wells in the area partially penetrate the aquifer. Therefore, the equation for partially penetrating wells applies unless (r) is greater than the aquifer thickness. In the latter instance, the equation for drawdown of a totally penetrating well (Theis formula) can be applied.

Analysis of the First Test

The thickness of the aquifer, as estimated from an analysis of data from previous tests in the same area, is about 700 feet (Hantush, 1961, p. 192). Therefore, for well No. 3 the aquifer thickness is less than (r), i.e., $r/b > 1$, and the Theis formula applies. In the instance of well No. 2 the aquifer thickness is greater than (r), i.e., $r/b < 1$, and the Hantush equation for partially penetrating wells is applied.

Analysis of data from well No. 2. The type curve method for partially penetrating wells is used (Hantush, 1961). The type-curve and observation data are shown in Figure 14. The calculated formation constants are

Thickness of the aquifer (b) = 727 feet

$K = 247$ gal/day/ft^2

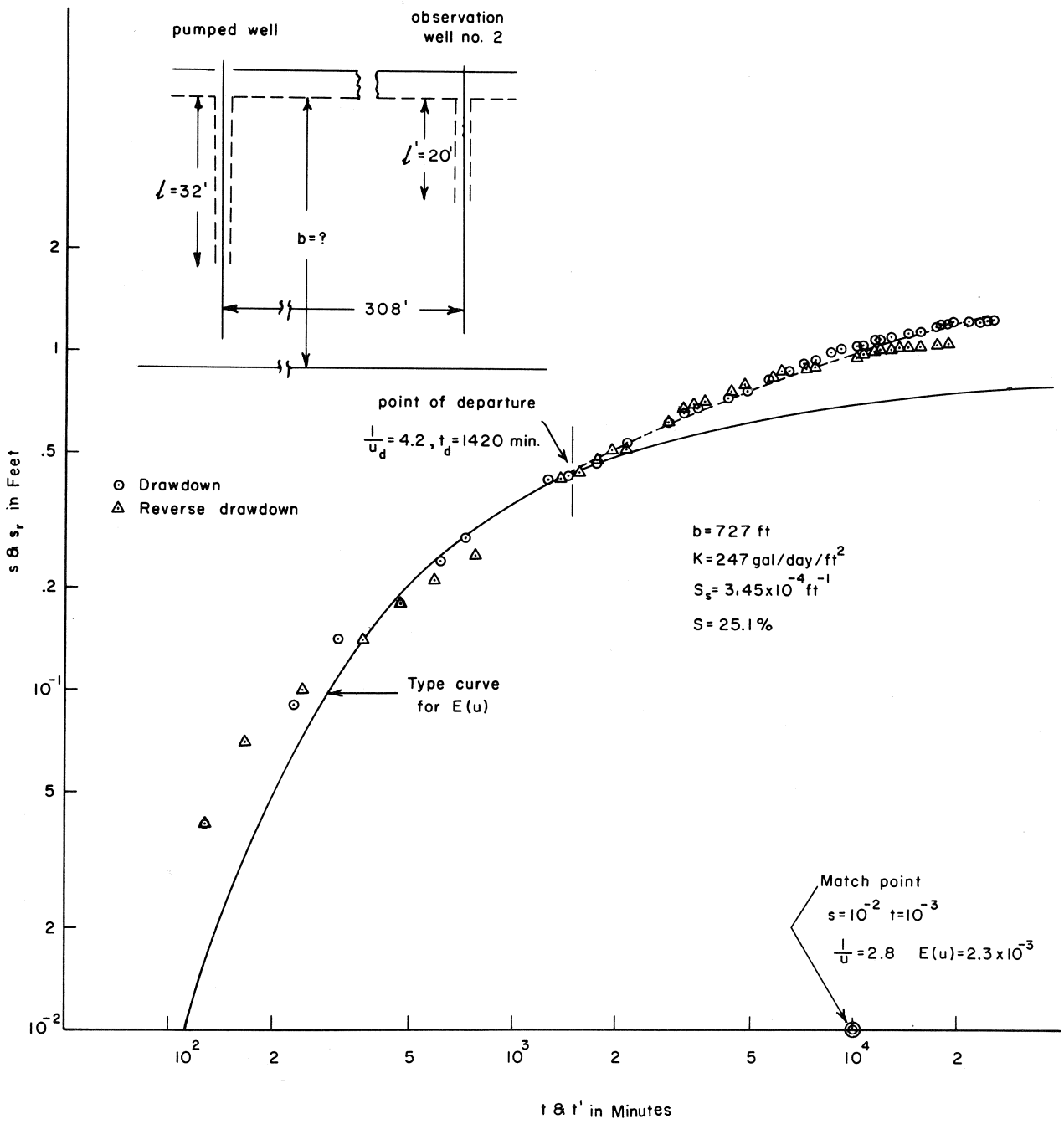


Fig. 14. Observed data and the theoretical type curve for well No. 2.

$$T = Kb = 180{,}000 \text{ gal/day/ft}$$

$$S_s = 3.45 \times 10^{-4} \text{ feet}^{-1}$$

$$S = 0.251$$

Analysis of data from well No. 3. The Theis type curve method is used (Todd, 1959). The type-curve and observation data are shown in Figure 15. The calculated formation constants are

$$T = 206{,}000 \text{ gal/day/ft}$$

$$S = 0.121$$

Analysis of the Second Test

Analysis of data from well No. 5. The well is partially penetrating the aquifer and the equation for the drawdown of the water table around a pumped well for small values of time is used (Hantush, 1960).

The drawdown versus time is plotted (Figure 16). The best fit straight line is passed through the observed points. By substituting the value of the slope of the straight line (m = 1/88.8) in equation 82, the value of the specific yield is found to be

$$S = 0.0041$$

The data available did not permit the calculation of other formation parameters.

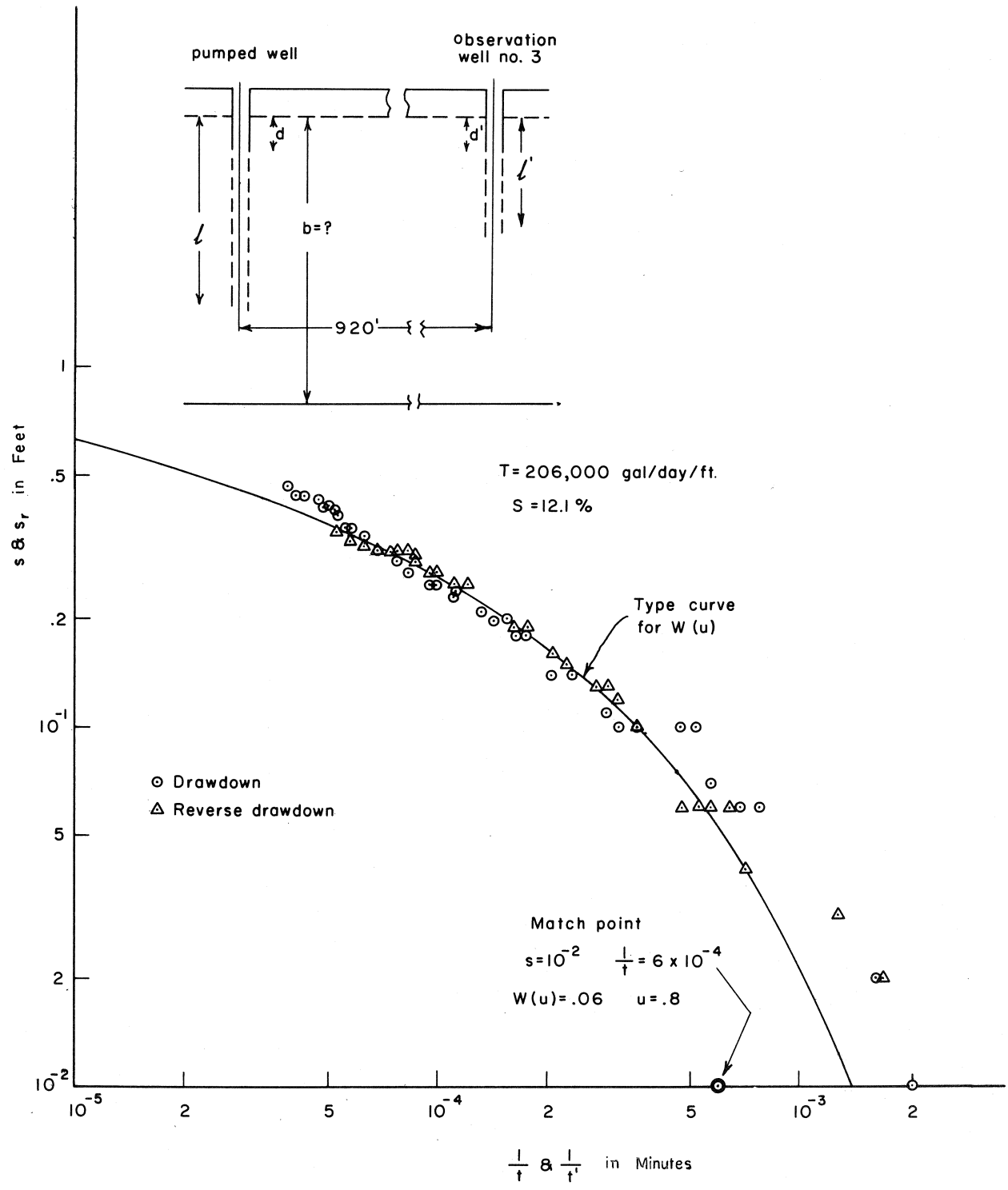
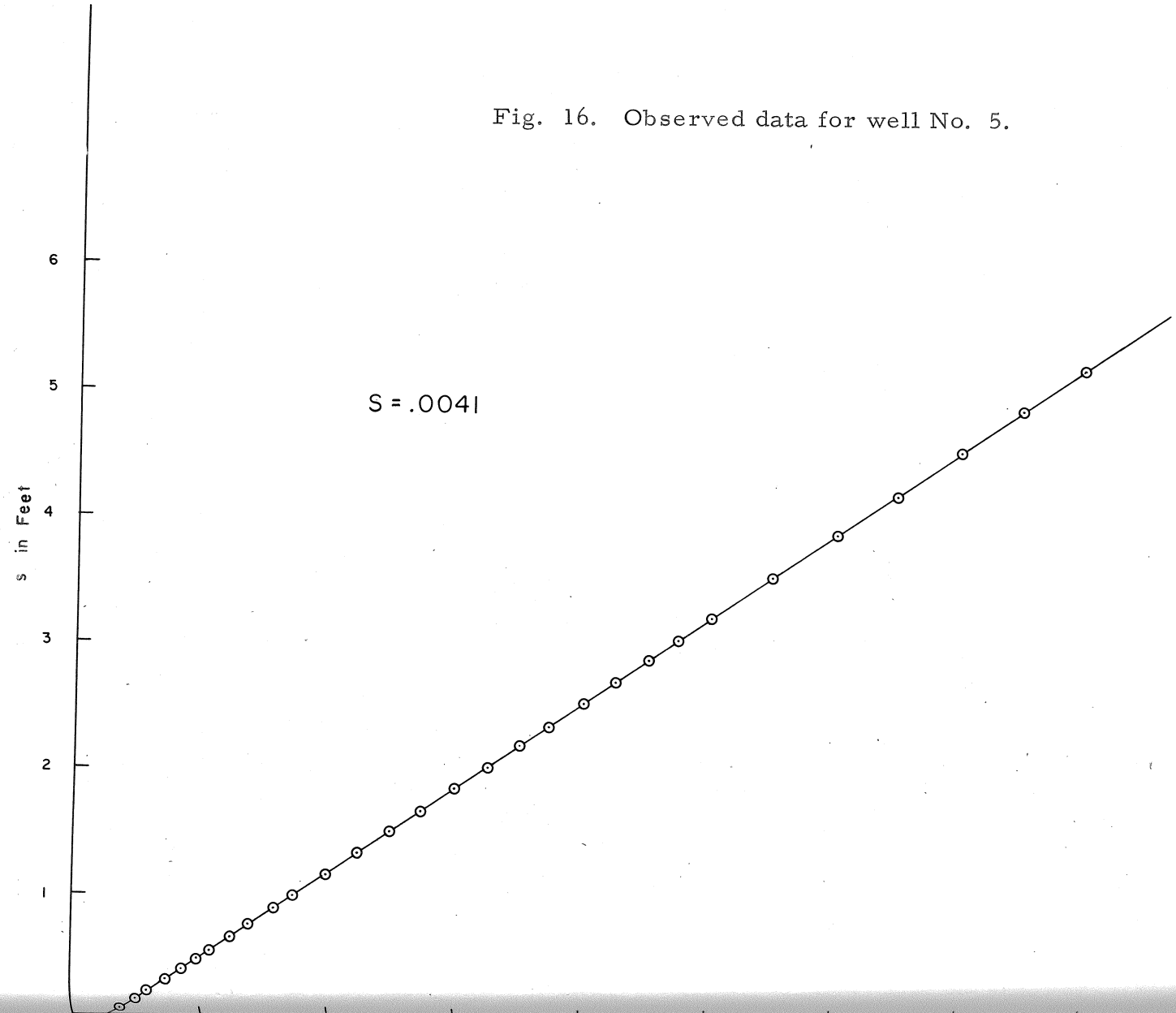


Fig. 15. Observed data and the theoretical type curve for well No. 3.

Fig. 16. Observed data for well No. 5.



COMPARISON BETWEEN SEISMIC AND HYDROLOGIC RESULTS

The values of porosity obtained from the different relationships between porosity and velocity for the locations near the pumping tests and the values of specific yield obtained from the pumping tests are listed in Table 1.

Location 1 lies at the east end of the refraction line between stations 94 and 105. These stations are 250 to 400 feet south of wells No. 3 and No. 2. The ranges of velocities at location 1 for the aerated and water-saturated layers are 2450 ft/sec to 1550 ft/sec and 5250 ft/sec to 5150 ft/sec, respectively.

Location 2 lies at the west end of the shot line between stations 20 and 24. These stations are 270 feet north of wells No. 4 and No. 5. The ranges of velocities at location 2 for the aerated and water-saturated layers are 2600 ft/sec to 3000 ft/sec and 6000 ft/sec to 5600 ft/sec, respectively.

In Table 1, the values of specific yield are always less than the values of porosity calculated from each relationship between velocity and porosity. This indicates that all the equations satisfy the condition that porosity (within the range of velocities considered) is greater than specific yield.

TABLE 1

Values of Specific Yield and Porosity at Locations 1 and 2

	Location 1 Stations 94-105	Location 2 Stations 20-24
Specific yield	0.121-0.251	0.0041-0.040
Porosity obtained from Wood's formula	0.388-0.44	0.250 -0.310
Modified Brandt's formula (ν_L = .5)	0.443-0.470	0.304 -0.365
Time-average formula (water saturated)	0.939-0.960	0.771 -0.855
Time-average formula (aerated)	0.430-0.705	0.340 -0.355
Equation (70)	0.545-0.485	0.35 -0.49
Equation (72)	0.585-0.55	0.40 -0.55
Time-average formula (aerated and water-saturated)	0.315-0.650	0.225 -0.240

To obtain a value of porosity from the hydrologic data, some values of specific retention had to be obtained. Eckis has published some data on the relation between porosity, specific yield, and specific retention (Figure 13) for an area in California (South Coastal Basin). The South Coastal Basin is geologically similar to the Rio Grande Valley near Socorro, New Mexico (Eckis, 1934). Specific retention values for the Socorro aquifer were obtained from his data. The procedure followed was to use Eckis's specific retention values for the specific yield values determined from the pumping tests. Table 2 is a summary of the specific retention obtained from Figure 13. These values assume that the distribution of grain sizes in the Socorro aquifers (for specific yield of 0.25 to 0.004) is the same as those in the South Coastal Basin aquifer. A check of the grain distribution of the aquifers at the site of the pumping tests indicates considerable similarity to the grain distributions in Figure 13.

TABLE 2

Porosity Values From Pumping Tests

Location	Aquifer constants	Values
Location 1	Specific yield	0.12 -0.25
	Specific retention for the above values of specific yield (obtained from Figure 13)	0.31 -0.18
	Porosity	0.43 -0.43
Location 2	Specific yield	0.004-0.04
	Specific retention for the above values of specific yield (obtained from Figure 13)	0.44 -0.40
	Porosity	0.44 -0.44

CONCLUSIONS

All the equations used in calculating the value of porosity from the compressional wave velocity simplify the physical properties of the aquifers. The use of simplified formulas was necessary because the measurement of a single parameter, the velocity, permitted only the calculation of a single unknown, the porosity. Despite the simplifications, three of the relationships between velocity and porosity appear to yield reasonable values of porosity. From the values of specific retention obtained from Eckis figure (1934), the porosity at locations 1 and 2 was found to range from 0.43 to 0.44. On this basis, the values of porosity calculated from Wood's formula (equation (73)), modified Brandt's formula (equation (75)), and the formula relating the velocities of the aerated and water saturated layers (equation (70)), are reasonable. The range of porosity obtained from the three equations is 0.388 to 0.545. Equations (75) and (70) which do not appear in the literature were derived from the work of Brandt (Brandt, 1955) and Biot (1956 a,b).

If more than two pumping tests had been conducted, more values for the aquifer constants could have been obtained and a better evaluation of the velocity-porosity relationships could have been made. However,

on the basis of the limited hydrological data, the three velocity-porosity relationships mentioned above do appear to give reasonable values of porosity.

APPENDIX

List of Symbols

C_b — Rock bulk compressibility, the fractional change in total or bulk volume of the porous rock per unit change in uniform pressure;

C_L — Liquid compressibility, the fractional change in the liquid volume per unit change in uniform pressure;

C_p — Pore compressibility, the fractional change in pore volume per unit change in uniform pressure;

C_r — Rock matrix compressibility, the fractional change in the volume of the solid rock material (without pores) per unit change in uniform pressure;

$\overline{C}$ — Constant (approximately equal to one);

$E(u)$ — $M(u, \frac{l}{2}) - M(u, \frac{d}{2})$ tabulated functions;

$\overline{K}$ — Constant (depends on the type of formation);

P — Pressure on outside of enclosed aggregates of particles;

P_L Pressure of liquid;

S_s Specific storage (volume of water released from storage by a unit volume of the aquifer under a unit head decline);

V_d Compressional wave velocity in the aerated (dry) layer;

V_w Compressional wave velocity in the water-saturated layer;

ϕ Porosity (ratio of the volume of voids between the grains of sedimentary sample to the total volume of the sediment);

ν Poisson's ratio for the water-saturated layer;

ν' Poisson's ratio for the aerated layer;

ν_L Poisson's ratio of liquid-saturated aggregate of particles.

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